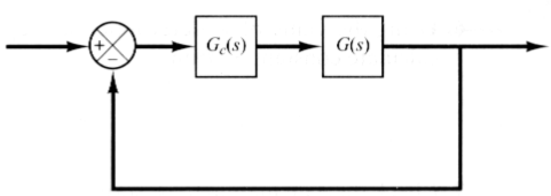
Control System Design: *Lag-Lead Compensator*

Control system diagram in unity feedback

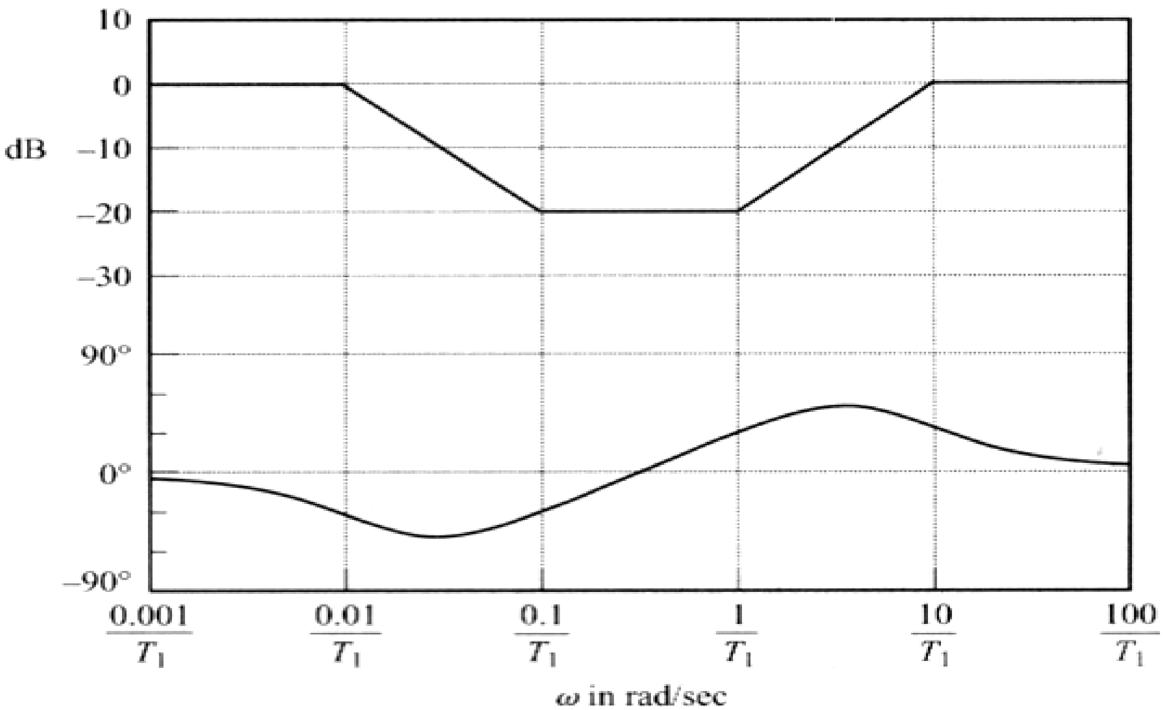


$G_C(s)$ – Compensator / Controller; *$G(s)$* – Plant / Transfer function

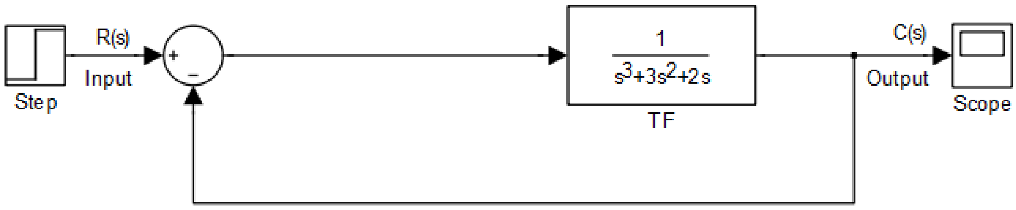
Lag-Lead compensation techniques based on the frequency response approach

Lag-Lead compensator transfer function

$$G_c(s) = K_c\left(\frac{(T_1 s + 1)(T_2 s + 1)}{\left(\frac{T_1}{\beta} s + 1\right)(\beta T_2 s + 1)}\right) = K_c\left(\frac{\left(s + \frac{1}{T_1}\right)\left(s + \frac{1}{T_2}\right)}{\left(s + \frac{\beta}{T_1}\right)\left(s + \frac{1}{\beta T_2}\right)}\right);\ \ \beta > 1$$



Example **Lag-**Lead design; Desired system is K_v of 10 sec^{-1}, PM is at least 50° and GM is at least 10 dB



Determine and analysis of previous information

Open-loop TF is

; Type_______

Closed-loop TF is

Closed-loop poles are ____

Bandwidth frequency (ω_{BW}) = ____________rad/sec

Gain margin(GM) =______________ dB; Phase margin(PM) = ________________ degree

Static velocity error constant (K_v) = __________________ sec^{-1}

Settling time = _______________________________sec (5% error)

Step I: Determine total gain (K) of open-loop TF to satisfy the requirement on the given static velocity error constant (K_v)= 20

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = \lim_{s\to 0} sG_c(s)\left(\frac{1}{(s+1)(s+2)s}\right) = 10$$

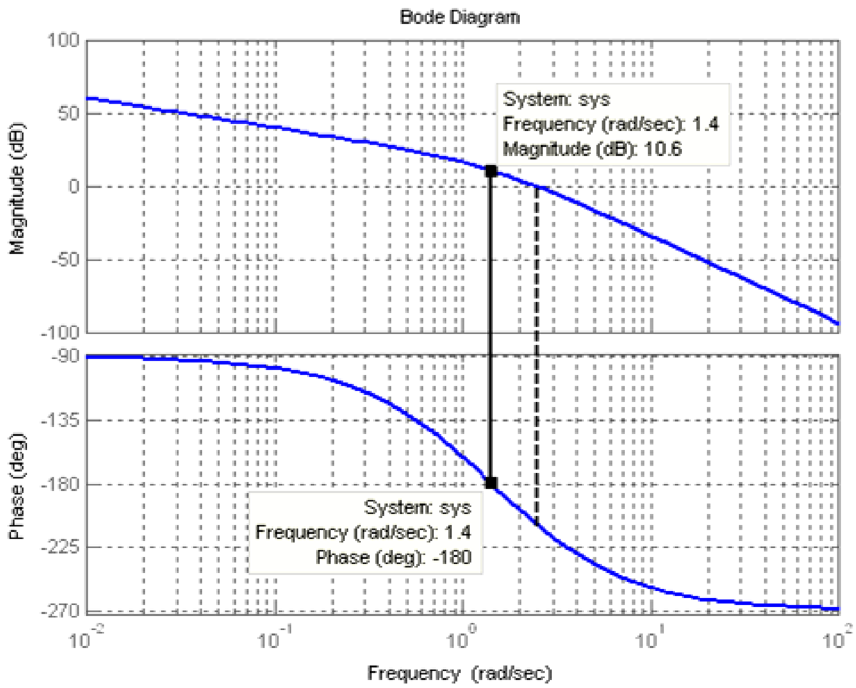
where $K_c = K$, thus

$$K = 10(2) = 20 \rightarrow K = 20$$

New open-loop transfer function

$$G_0(s) = \frac{20}{(s+1)(s+2)s} = \frac{20}{s^3+3s^2+2s}$$

Step II: Plot bode diagram of open-loop TF with new gain such as



Unstable ;Phase margin(PM)= ____deg. at ____ rad/sec; Gain margin(GM)= =+10.4 dB at 10.6 rad/sec

Step III: Design phase-lag part; Select new gain crossover frequency $\omega = 1.4 \text{ rad/sec}$

Thus, corner frequency of phase-lag part(zero part) $\omega = 0.14 \text{ rad/sec}$

$$\frac{1}{T_2} = 0.14 \rightarrow T_2 = 7.143$$

Step IV: Determine , $\sin \varphi_{max} = \frac{1-\alpha}{1+\alpha}$, where $\alpha = 1/\beta$

$$sin\varphi_{max} = \frac{\beta - 1}{\beta + 1}$$

Step V: From $-20\log\frac{1}{\sqrt{\alpha}} = -20\log\frac{1}{\sqrt{0.198}} = -7.028 \text{ dB}$ at ω_{max}

Select ω_{max} to be new phase crossover frequency at this frequency must be – 7.028 dB. At 9.37 rad/sec is ω_{max}

$$\omega_{max} = \frac{1}{T\sqrt{\alpha}} = 9.37 \quad rad/sec$$

Thus T= 0.16; $\alpha = 0.198$

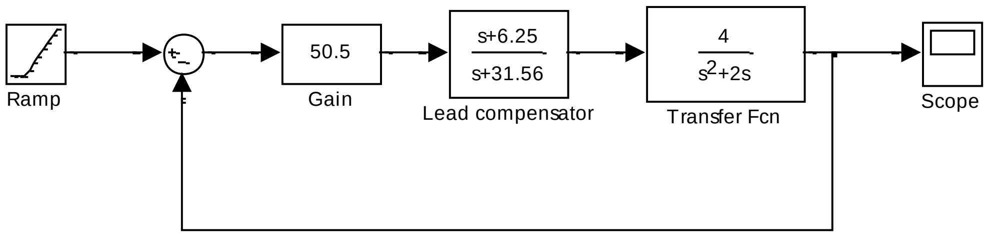
Now lead compensator is

$$G_c(s) = K_c \frac{s + 1/T}{s + 1/\alpha T} = K_c \left(\frac{s + 6.25}{s + 31.56}\right)$$

<u>Step VI:</u> Determine gain of lead compensator

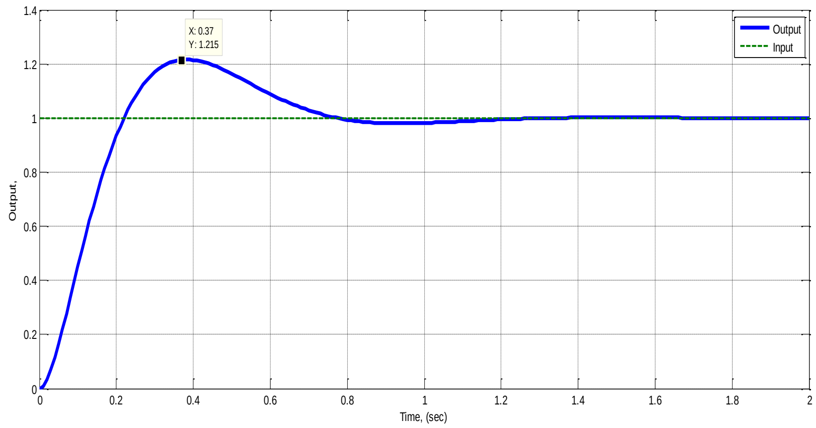
$$K_c\alpha = K = 10 \rightarrow K_c = 50.5$$

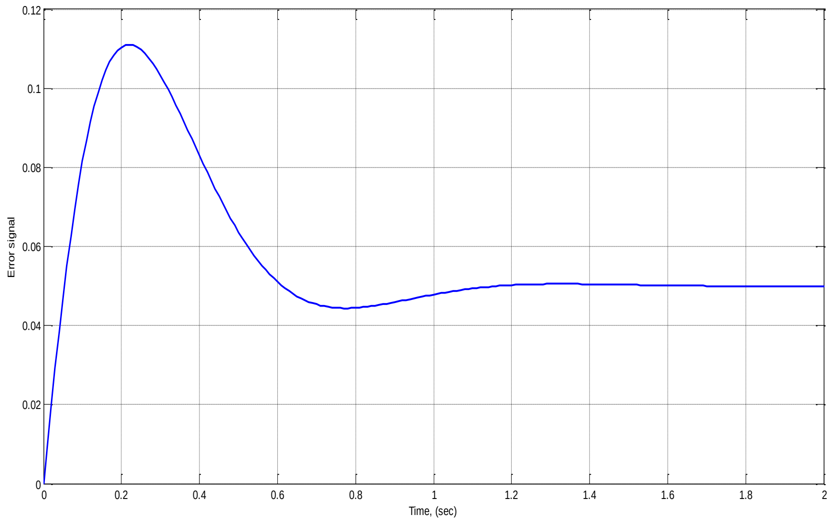
Now lead compensator is $G_c(s) = 50.5\left(\frac{s+6.25}{s+31.56}\right)$



Check steady state error for unit-ramp input relation with static velocity error constant and PM relation with damping ratio (% overshoot)

- Steady state error for unit-ramp input is 0.05 (Static velocity error constant is 20 sec^{-1})
- **% overshoot is 12.15** ....





Bode diagram of Open-loop TF $G_c(s)G(s) = 50.5\left(\frac{s+6.25}{s+31.56}\right)\left(\frac{4}{s(s+2)}\right)$

Phase margin(PM)= ________ deg. at ______ rad/sec ; Gain margin(GM)=_____ dB at _____ rad/sec